# OXYGEN TRANSFER IN HIGH-SPEED FERMENTER UNDER STEADY CULTIVATION CONDITIONS 

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#### Abstract

A method for volume mass transfer coefficient calculation under steady fermentation conditions, including correction of partial oxygen pressure in bubbles, has been developed. Number of continuous cultivations on ethanol and sulphite substrates were evaluated on the basis of the suggested method.


Oxygen transfer in continuous aerobic fermenters, where both process and cultivations parameters are slowly changed , may be considered as steady. Then molar oxygen transfer rate $\dot{M}_{\mathrm{O} 2, \mathrm{vo}}$ is given by correlation of Lewis and Whitman (e.g. ref. ${ }^{1}$ )

$$
\begin{equation*}
\dot{M}_{\mathrm{O} 2, \mathrm{vo}}=3600 k_{\mathrm{L}} a\left(C^{*}-C_{\mathrm{L}}\right) . \tag{1}
\end{equation*}
$$

Actual equilibrium concentration of dissolved oxygen $\bar{C}^{*}$ can be found experimentally with great difficulties only. Therefore pressumptions concerning bubble passage through aerated system used to be set for $\bar{C}^{*}$ determination. Mostly a presumption of "piston flow" (ref. ${ }^{2}$ ) or "ideal mixing" (refs ${ }^{3-5}$ ) of bubbles is suggested. Volume mass transfer coefficient $k_{\mathrm{L}} a$ obtained on the basis of both above mentioned pressumptions differ significantly, especially if $\dot{M}_{\mathrm{O2}, \mathrm{vo}}$ values are high ${ }^{6}$. Estimation of $\dot{M}_{\mathbf{O 2}, \mathrm{vo}}$ from oxygen balance in fermenter is not difficult and in most cases a negligible difference in the inlet and outlet air volumetric flow rate have been observed

$$
\begin{equation*}
\dot{V}_{\mathrm{G} 1} \doteq \dot{V}_{\mathrm{G} 2} . \tag{2}
\end{equation*}
$$

This conditions complies with sufficient accuracy for many fermentation processes, considering that oxygen reduction is compensed by carbon dioxide increase. Hence

$$
\begin{equation*}
\dot{M}_{\mathrm{O} 2, \mathrm{vo}}=3600\left(\dot{V}_{\mathrm{G} 1} / V_{\mathrm{O}}\right)\left(\varrho_{\mathrm{O} 2} / M_{\mathrm{O} 2}\right)\left(y_{1}-y_{2}\right) . \tag{3}
\end{equation*}
$$

Substituting the dimensionless constant for oxygen

$$
\begin{equation*}
k_{\mathrm{O} 2}=M_{\mathrm{O} 2} p / \varrho_{\mathrm{O} 2} k_{\mathrm{H}} \tag{4}
\end{equation*}
$$

and using Henry's law ${ }^{7}$

$$
\begin{equation*}
C^{*}=y p / k_{\mathrm{H}} \tag{5}
\end{equation*}
$$

Eq. (3) may be rewritten after introduction of equilibrium concentration of dissolved oxygen $C^{*}$

$$
\begin{equation*}
\dot{M}_{\mathrm{O} 2, \mathrm{vo}}=3600\left(\dot{V}_{\mathrm{G} 1} / V_{\mathrm{o}}\right)\left(C_{1}^{*}-C_{2}^{*}\right) / k_{\mathrm{O} 2} . \tag{6}
\end{equation*}
$$

The concentration $C^{*}$ may differ in various points of fermented broth - partly in consequence of oxygen decrease in bubbles, and partly because of hydrostatic pressure dependence on the height of batch.
Oxygen concentration in bubble is in relation to its molar fraction ${ }^{7}$

$$
\begin{equation*}
C_{\mathrm{B}}=y \varrho_{\mathrm{O} 2} / M_{\mathrm{O} 2}, \tag{7a}
\end{equation*}
$$

combining Eqs (5) and (6) gives

$$
\begin{equation*}
C_{\mathrm{B}}=C^{*} / k_{\mathrm{O} 2} . \tag{7b}
\end{equation*}
$$

Reduction of oxygen concentration in a bubble can be expressed, after relative hold-up $\Phi_{G}$ introduction, by the following relationships ${ }^{6}$

$$
\begin{gather*}
-\mathrm{d} C_{\mathrm{B}} / \mathrm{d} t=k_{\mathrm{L}} a\left(C^{*}-C_{\mathrm{L}}\right)\left(1-\Phi_{\mathrm{G}}\right) / \Phi_{\mathrm{G}}  \tag{8a}\\
-\mathrm{d} C^{*} / \mathrm{d} t=k_{\mathrm{O} 2} k_{\mathrm{L}} a\left(C^{*}-C_{\mathrm{L}}\right)\left(1-\Phi_{\mathrm{G}}\right) / \Phi_{\mathrm{G}} \tag{8b}
\end{gather*}
$$

To be able to solve the relationship ( $8 b$ ) the following simplifying assumptions have been made: A1 - the system is monodisperse and non-coalescent, A2 - the system is homogeneous from point of view of specific interfacial area and relative hold-up, A3 - rising velocity of bubbles is constant ( $w_{B}=$ const.), A4 - dissolved oxygen concentration $C_{\mathrm{L}}$ is proportional to $C^{*}$ at the given point of batch $\left(C_{\mathrm{L}} / C^{*}=k_{\mathrm{C}}\right)$, A5 - mass transfer coefficient is constant ( $k_{\mathrm{L}}=$ const.). Using assumption A4 and Eq. (8b) gives

$$
\begin{equation*}
-\mathrm{d} C^{*} / \mathrm{d} t=k_{\mathrm{O} 2}\left(1-k_{\mathrm{C}}\right) k_{\mathrm{L}} a C^{*}\left(1-\Phi_{\mathrm{G}}\right) / \Phi_{\mathrm{G}} \tag{9}
\end{equation*}
$$

replacing $C^{*}$, in accordance with Henry's law, by variables $y, p$ and differentiation Eq. (5), with respect to time gives

$$
\begin{equation*}
\mathrm{d} C^{*} / \mathrm{d} t=\left(\frac{p \mathrm{~d} y}{\mathrm{~d} t}+\frac{y \mathrm{~d} p}{\mathrm{~d} t}\right) / k_{\mathrm{H}} \tag{10}
\end{equation*}
$$

The pressure at a given point of batch can be expressed by means of depth of level $h$ (related to liquid filling only) and the pressure above the level $\boldsymbol{p}_{2}$

$$
\begin{gather*}
p=p_{2}+\varrho_{\mathrm{L}} g\left(H_{\mathrm{O}}-h\right)  \tag{11}\\
\mathrm{d} p=-\varrho_{\mathrm{L}} g \mathrm{~d} h . \tag{12}
\end{gather*}
$$

The residence time $t$ of a bubble, if assumption A3 and depth of level are used, is given by

$$
\begin{gather*}
w_{\mathrm{B}}=H_{\mathrm{O}} \dot{V}_{\mathrm{G} 1}\left(1-\Phi_{\mathrm{G}}\right) / V_{\mathrm{O}} \Phi_{\mathrm{G}}  \tag{13}\\
t=h / w_{\mathrm{B}}  \tag{14}\\
\mathrm{~d} t=\frac{\Phi_{\mathrm{G}} \cdot V_{\mathrm{O}}}{H_{\mathrm{O}} \dot{V}_{\mathrm{G} 1}\left(1-\Phi_{\mathrm{G}}\right)} \mathrm{d} h \tag{15}
\end{gather*}
$$

Combining Eqs (11), (12), (10), (15), and (9) gives

$$
\begin{equation*}
-\frac{\mathrm{d} y}{y}=\left[k_{\mathrm{L}} a \frac{k_{\mathrm{O} 2} V_{\mathrm{O}}}{H_{\mathrm{O}} \dot{V}_{\mathrm{G} 1}}\left(1-k_{\mathrm{C}}\right)-\frac{\varrho_{\mathrm{L}} g}{p_{2}+\varrho_{\mathrm{L}} g\left(H_{\mathrm{O}}-h\right)}\right] \mathrm{d} h \tag{16}
\end{equation*}
$$

Considering boundary conditions

$$
\begin{array}{ll}
h_{\mathrm{o}}=0, & y=y_{1}  \tag{17a}\\
h=H_{\mathrm{o}}, & y=y_{2}
\end{array}
$$

and introducing inlet pressure

$$
\begin{equation*}
p_{1}=p_{2}+\varrho_{\mathrm{L}} g H_{\mathrm{O}} \tag{17b}
\end{equation*}
$$

yields

$$
\begin{equation*}
k_{\mathrm{L}} a=\frac{\dot{V}_{\mathrm{G} 1} / V_{\mathrm{O}}}{k_{\mathrm{O} 2}\left(1-k_{\mathrm{C}}\right)} \ln \left(y_{1} p_{1} / y_{2} p_{2}\right) \tag{18}
\end{equation*}
$$

Considering the balance value of $\dot{M}_{\mathrm{O} 2, \mathrm{vo}}$ and rearrangement of Eq. ( 6 ) yields

$$
\begin{equation*}
\dot{V}_{\mathrm{G} 1} / V_{0}=\frac{\dot{M}_{\mathrm{O} 2, \mathrm{vo}}}{3600\left[\left(C_{1}^{*}-C_{2}^{*}\right) / k_{\mathrm{O} 2}\right]} \tag{19}
\end{equation*}
$$

Defining equilibrium concentration $C_{A T}^{*}$ which corresponds to solubility of oxygen from air under atmospheric conditions

$$
\begin{equation*}
C_{1}^{*}=C_{\mathrm{AT}}^{*}\left(p_{1} / p_{\mathrm{AT}}\right)\left(y_{1} / 0 \cdot 21\right) \tag{20}
\end{equation*}
$$

gives finally

$$
\begin{equation*}
k_{\mathrm{L}} a=\frac{\dot{M}_{\mathrm{O} 2, \mathrm{vo}} \ln \left(y_{1} p_{1} / y_{2} p_{2}\right)}{3600 C_{\mathrm{AT}}^{*}\left(y_{1} p_{1} / 0 \cdot 21 p_{\mathrm{AT}}\right)\left(1-k_{\mathrm{C}}\right)\left(1-y_{2} p_{2} / y_{1} p_{1}\right)} \tag{21}
\end{equation*}
$$

The volume mass transfer coefficient depends tightly on specific liquid-gas interfacial area. Influence of mixing and aeration on $k_{\mathrm{L}} a$ has been studied intensively recently ${ }^{8-24}$; results mostly presented in form of empirical correlations

$$
\begin{gather*}
k_{\mathrm{L}} a=f\left(P_{\mathrm{G}} / V_{\mathrm{O}}, w_{\mathrm{F}}, \Phi_{\mathrm{G}}\right)  \tag{22}\\
\left(k_{\mathrm{L}} a / w_{\mathrm{F}}\right)\left(v^{2} / g\right)^{1 / 3}=f\left(P_{\mathrm{G}} / V_{\mathrm{O}} \varrho_{\mathrm{L}} w_{\mathrm{F}} g\right) \tag{23}
\end{gather*}
$$

Generalisation of results of different authors, however, is not easy owing to different experimental methods, different ways of $k_{\mathrm{L}} a$ evaluation and large range of physical, chemical and electrochemical characteristics of solutions.

## EXPERIMENTAL

Oxygen transfer was measured in two fermenters of volume 0.05 and $1 \mathrm{~m}^{3}$. Both fermenters were equipped with high-speed, bottom driven agitators. The set-up scheme has already been described ${ }^{6,25}$ as well as agitators and baffles sizes ${ }^{25}$.

Scheme of measuring and automatic control systems of both fermenters is shown in Fig. 1. Regarding the fact that cultivations were carried out under steady experimental conditions, the fermenters were equipped by regulation of temperature, pH , filling volume, continuous dosing of all components and continuous measurement of all necessary parameters: $y_{\mathrm{CO}_{2}}, y_{2}$, $C_{\mathrm{L}, \mathrm{O}_{2}}, \dot{V}_{\mathrm{G} 1}, M_{\mathrm{K}}, n, V_{\mathrm{O}}$. The values of $\dot{M}_{\mathrm{O} 2, \mathrm{vo}}$ and $k_{\mathrm{L}} a$ have been calculated using Eq. (3) and Eq. (2I) respectively, from experimental data.

Power input calculation of agitators has been based on torsional moment (Research Institute of Chemical Equipment tensometric dynamometer) and rotational speed after substractions of all passive losses of driving system

$$
\begin{equation*}
P_{\mathbf{G}}=2 \pi n\left(M_{\mathbf{K}}-M_{\mathrm{K}, \mathrm{O}}\right) . \tag{24}
\end{equation*}
$$

Fig. 1
Scheme of measurement and controll circuits. 1 Rotational speed measurement, 2 torque measurement, 3 dissolved oxygen level measurement, 4 temperature measurement and control, 5 pH measurement and control, 6 ammonia water flow measurement, 7 oxygen concentration in gas measurement, $8 \mathrm{CO}_{2}$ concentration measurement, 9 batch volume measurement, 10 liquid level measurement, 11 air flow measurement


Two technologies of animal feed protein production were utilized in the course of our experiments:
a) Research Institute of Food Industry, Prague, technology using Torulopsis ethanoli toleranse yeast and ethanol substrate,
b) Institute of Microbiology, Czechoslovak Academy of Sciences, Prague, technology using Candida utilis yeast and $\mathbf{M g}$-bisulphite leach.

Continuous cultivations were in progress c. $70-90 \mathrm{~h}$.
Oxygen balance calculation was based on turbine flow meter and outlet gas analysis data, measured by Permolyt and Infralyt Junkalor Dessau Analysers. Dissolved oxygen level was traced by Ingold sensor.

The equilibrium concentration of dissolved oxygen $C^{*}$ was determined, after sensor calibration in distilled water, by direct measurement in fermented medium, close upon temperature and pH adjustment to the operating conditions: ethanol broth ( $\mathrm{pH} 4 \cdot 2$ ), $C_{\mathrm{AT}}^{*}=0.222 \mathrm{~mol} \mathrm{~m}^{-3}$ and sulphite broth ( pH 5.4 ),$C_{\mathrm{AT}}^{*}=0.220 \mathrm{~mol} \mathrm{~m}^{-3}$.

## RESULTS

The quantities $k_{\mathrm{L}} a, P_{\mathrm{G}} / V_{\mathrm{O}}$, and $w_{\mathrm{F}}$ were evaluated during fermentation experiments. To find the effect of mixing and aeration on $k_{\mathrm{L}} a$ a regression of the following form was investigated

$$
\begin{equation*}
k_{\mathrm{L}} a=k_{1}\left(P_{\mathrm{G}} / V_{\mathrm{O}}\right)^{a_{1}} w_{\mathrm{F}}^{\boldsymbol{a}_{2}} \tag{25}
\end{equation*}
$$

separately for ethanol and sulphite fermentation experiments.
Results of one experiment in the fermenter of volume $0.05 \mathrm{~m}^{3}$ and two in the fermenter of volume $1 \mathrm{~m}^{3}$ were used for ethanol broth correlation

$$
\begin{equation*}
k_{\mathrm{L}} a=0,364\left(P_{\mathrm{G}} / V_{\mathrm{O}}\right)^{0.47} w_{\mathrm{F}}^{0.64} \pm 0.027 \tag{26}
\end{equation*}
$$

with range of validity $0.42 \leqq P_{\mathrm{G}} / V_{\mathrm{O}}\left(\mathrm{kW} \mathrm{m}^{-3}\right) \leqq 6.06$ and $0.068 \leqq w_{\mathrm{F}}\left(\mathrm{cms}^{-1}\right) \leqq$ $\leqq 0.810$. Residual variance was 0.035 , correlation coefficient equaled to 0.785 . Molar oxygen transfer rates were within the range 36 and $278 \mathrm{~mol} \mathrm{~m}^{-3} \mathrm{~h}^{-1}$. In spite of relatively small value of correlation coefficient and greater variance of measurement the effect of both regression variables was statisticaly important. The variance of measurement is also obvious from comparative diagram (see Fig. 2) which also demonstrates absence of systematic deviation in both sets of experimental data.

In a similar way results of three cultivation experiments with sulphite leach in $0.05 \mathrm{~m}^{3}$ fermenter were processed by regression analysis method and the following relationship has been obtained

$$
\begin{equation*}
k_{\mathrm{L}} a=0.207\left(P_{\mathrm{G}} / V_{\mathrm{O}}\right)^{0.44} w_{\mathbf{F}}^{0.67} \pm 0.029 \tag{27}
\end{equation*}
$$

with range of validity $0.91 \leqq P_{\mathrm{G}} / V_{\mathrm{O}}\left(\mathrm{kW} \mathrm{m}^{-3}\right) \leqq 5.22$ and $0.085 \leqq w_{\mathrm{F}}\left(\mathrm{cm} \mathrm{s}^{-1}\right) \leqq$ $\leqq 0.276$. Residual variance and correlation coefficient were 0.00082 and 0.968 , respectively. Molar oxygen transfer rates were within the range 29 and 122 mol . . $\mathrm{m}^{-3} \mathrm{~h}^{-1}$. Correlation coefficient is considerably greater than it has been in the previous correlation (26). Fig. 3 also demonstrates significantly lower variance of measurement in comparison with the value calculated for fermentation on ethanol substrate.

## DISCUSSION

The found exponents of correlation variables $P_{\mathrm{G}} / V_{\mathrm{O}}$ and $w_{\mathrm{F}}$ in Eqs (26) and (27) have agreed well with data published in literature ${ }^{8,11-13,24,26,27}$. Both the results of data evaluation and the air hold-up values have shown that just forming interfacial area is different for various fermentation media, what is in good agreement with the paper of Bylinkina ${ }^{27}$. The values of $k_{\mathrm{L}} a$ were, under the same conditions of mixing and aeration, significantly greater for ethanol broth. Visual observation of both processes showed a higher "foaming" and greater relative air hold-up for sulphite technology. Explanation may be found probably partly in smaller mass transfer coefficient $k_{\mathrm{L}}$, and partly in increased mean bubble diameter in the fermenta-


Fig. 2
Diagram comparing $k_{\mathrm{L}} a$ data - experimental and calculated by Eq. (26) for ethanol broth. - Fermenter $0.05 \mathrm{~m}^{3}$, fermenter $1 \mathrm{~m}^{3}$


Fig. 3
Diagram comparing $k_{\mathrm{L}} a$ data - experimental and calculated by Eq. (27) for sulphite broth

Table I
Evaluation of $k_{\mathrm{L}} a$ for variable $\dot{M}_{\mathrm{O} 2, \text { vo }}$ in fermenter $0.05 \mathrm{~m}^{3}$ at $60{\dot{V_{\mathrm{G}}}} / V_{\mathrm{O}}=0.75 \mathrm{~min}^{-1}$, $V_{\mathrm{O}} / V_{\mathrm{C}}=0.5$

| $\dot{M}_{\mathrm{O2}, \mathrm{vO}}$ <br> $\mathrm{mol} \mathrm{m}^{-3}$ <br> $\mathbf{h}^{-}$ | $k_{\mathrm{L}} a^{a}$ <br> $\mathbf{s}^{-1}$ | $k_{\mathrm{L}} a^{b}$ <br> $\mathbf{s}^{-1}$ | $\delta$ <br> $\%$ |
| :---: | :---: | :---: | :---: |
| 100 | 0.163 | 0.143 | 14.0 |
| 150 | 0.296 | 0.232 | 27.5 |
| 200 | 0.475 | 0.340 | 39.7 |

${ }^{a}$ Calculated under IM pressumption; ${ }^{b}$ calculated by Eq. (21).

Table II
Evaluation of $k_{\mathrm{L}} a$ for variable aeration in fermenter $0.05 \mathrm{~m}^{3}$ at $\dot{M}_{\mathrm{O} 2, \mathrm{vO}}=100 \mathrm{~mol} \mathrm{~m}^{-3} \mathrm{~h}^{-1}$, $V_{\mathrm{O}} / V_{\mathrm{C}}=0.5$

| $60 \dot{V}_{\mathrm{G}} / V_{\mathrm{o}}$ <br> $\min ^{-1}$ | $k_{\mathrm{L}} a^{a}$ <br> $\mathbf{s}^{-1}$ | $k_{\mathrm{l}} a^{b}$ <br> $\mathbf{s}^{-1}$ | $\boldsymbol{\delta}$ <br> $\%$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.50 | 0.197 | 0.155 | 27.1 |
| 0.75 | 0.163 | 0.143 | 14.0 |
| 1.00 | 0.154 | 0.138 | 11.9 |

${ }^{a, b} C f$. Table 1 .

## Table III

Evaluation of $k_{\mathrm{L}} a$ for variable fermenter size at $60 \dot{V}_{\mathrm{G}} / V_{\mathrm{O}}=0.75 \mathrm{~min}^{-1}, \dot{M}_{\mathrm{O} 2, \mathrm{vO}}=150 \mathrm{~mol}$. $. \mathrm{m}^{-3} \mathrm{~h}^{-1}, V_{\mathrm{o}} / V_{\mathrm{C}}=0.5$

| $V_{\mathrm{C}}$ <br> $\mathrm{m}^{3}$ | $k_{\mathrm{L}} a^{a}$ <br> $\mathbf{s}^{-1}$ | $k_{\mathbf{L}} a^{b}$ <br> $\mathbf{s}^{-1}$ | $\delta$ <br> $\%$ |
| :---: | :---: | :---: | :---: |
| 0.05 0.296 0.232 28 <br> 1.5 0.279 0.227 23 <br> 50 0.247 0.213 16 |  |  |  |

${ }^{a}$ Calculated under IM pressumption including correction of partial pressure $\mathrm{O}_{2}$ on inlet air conditions; ${ }^{b}$ calculated by Eq. (2l).
tion medium as it follows from relationship between $\Phi_{G}, a$, and $d_{\mathrm{B}}$ (see ref. ${ }^{9}$ )

$$
\begin{equation*}
a \cong\left(1-\Phi_{\mathrm{G}}\right) / d_{\mathrm{B}} \tag{28}
\end{equation*}
$$

A reason of substantially greater deviations of ethanol broth experimental result in comparison to that obtained in sulphite fermented medium, can be explained by deviations from originally made pressumptions: greater fluctuation of bio-mass concentration in fermenter, greater deviation of $\dot{V}_{G 1}$ from $\dot{V}_{G 2}$ values, closer linkage to so-called Monod model of micro-organism growth ${ }^{26}$ (i.e. existence of feedback oxygen transfer on dissolved oxygen| level and rate of yeast growth) and the effect of change of ethanol concentration in fermentation medium on mass transfer coefficient and interfacial area size.

Comparison of our experimental $k_{\mathrm{L}} a$ and published data is rather questionable since $k_{\mathrm{L}} a$ magnitude depends on experimental methodology ${ }^{1,28}$, dispersion character (e.g. from point of view of coalescence ${ }^{14}$ ) and the method of determination of mean value $\bar{C}^{*}$. Effect of the last mentioned factor is evaluated on the basis of comparison of $k_{\mathrm{L}} a$ values, calculated under steady fermentation conditions and pressumption of ideal mixing of air (IM) and values calculated by Eq. (21) in Tables I, II, III.

Data in Table I show independently, in substance, the effect of specific power input of agitator ( $w_{F}=$ const.) on $k_{\mathrm{L}} a$ value. It may be concluded, on the basis of deviation $\delta$ increase, that using of pressumption IM brings about a substantial increasing of exponent $a_{1}$ in Eq. (25) in comparison with Eq. (21).

Data in Table II indicate, to a certain extent, the effect of increase of fermentation broth aeration on $k_{\mathrm{L}} a\left(P_{\mathrm{G}} / V_{\mathrm{O}}\right.$ value need not be entirely constant). With regard to the fact that aeration increase (by means of $60 \dot{V}_{\mathrm{G}} / V_{\mathrm{O}}$ here) decreases the deviation, we may assume that using of IM pressumption leads rather to exponent $a_{2}$ decrease in Eq. (25) in comparison with Eq. (21).

Table III exhibits the effect of scale-up on $k_{\mathrm{L}} a$ deviation. The values of $k_{\mathrm{L}} a$ decrease with increasing total volume $V_{C}$ in both "models". This decrease is more significant for IM pressumption and the decreasing effect of mixing, represented by the exponent $a_{1}$ in Eq. (25), may not be neglected during scaling-up.
Comparison of both $k_{\mathrm{L}} a$ evaluation methods shows the possibility of origin of significant differences depending on the type of correlation (25). If relationships published in literature are used, it is necessary to adhere to the conditions under which the relationship has been derived.

## LIST OF SYMBOLS

| $a$ | specific interfacial area, $\mathrm{m}^{-1}$ |
| :--- | :--- |
| $a_{1,2}$ | exponents in Eq. (25) |
| $C$ | molar concentration, $\mathrm{mol} \mathrm{m}^{-3}$ |
|  |  |
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| $d_{\mathrm{B}}$ | mean bubble diameter, m |
| :---: | :---: |
| $g$ | gravitational constant, $\mathrm{m} \mathrm{s}^{-2}$ |
| $H_{0}$ | height of liquid level, m |
| $h$ | height coordinate, $m$ |
| $k_{1}$ | constant in Eq. (25) |
| $k_{\text {C }}$ | relative concentration of dissolved oxygen, - |
| $k_{\text {H }}$ | Henry's constant, Pa m ${ }^{3} \mathrm{~mol}^{-1}$ |
| $k_{\text {L }}$ | mass transfer coefficient, $\mathrm{m} \mathrm{s}^{-1}$ |
| $k_{\text {O2 }}$ | constant for oxygen (see Eq. (4)) |
| $\dot{M}_{\mathrm{O} 2}$, vo | rate of molar oxygen transfer, mol m ${ }^{-3} \mathrm{~h}^{-1}$ |
| $M_{\mathrm{O} 2}$ | molar weight of oxygen, $\mathrm{kg} \mathrm{mol}^{-1}$ |
| $M_{\text {K }}$ | torsional moment, Nm |
| $n$ | agitator revolutions, $\mathrm{s}^{-1}$ |
| $P_{\text {G }}$ | agitator power input in aerated system, W |
| $p$ | pressure, Pa |
| $t$ | time, s |
| $\dot{V}_{G}$ | volumetric air flow rate, $\mathrm{m}^{3} \mathrm{~s}^{-1}$ |
| $V_{0}$ | volume of liquid filling, $\mathrm{m}^{3}$ |
| $V_{C}$ | total volume filling, $\mathrm{m}^{3}$ |
| $w$ | flow velocity, $\mathrm{ms}^{-1}$ |
| $y$ | molar fraction of gas (without designation for oxygen) |
| $v$ | kinematic viscosity of liquid, $\mathrm{m}^{2} \mathrm{~s}^{-1}$ |
| 0 | density, $\mathbf{k g ~ m}^{-3}$ |
| $\Phi_{G}$ | relative air hold-up |

Subscripts

| 1 | inlet |
| :--- | :--- |
| 2 | outlet |
| AT | under atmospheric pressure |
| B | bubbles |
| $\mathrm{CO}_{2}$ | carbon dioxide |
| G | gas |
| F | fictive, related to cross section |
| L | liquid |
| O 2 | oxygen |
|  | equilibrium value |

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